The Brindel Transformation

A Fixed-Point Method in Analytic Number Theory

From the Zeta Function to Automorphic L-Functions

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Abstract

This document explains, step by step, the discovery and application of the Brindel transformation: the observation that for any integer $n \geq 2$, there exist complex numbers s_n satisfying $n^{s_n} = n$. We show how this simple fixed-point equation, when applied to the Riemann zeta function $\zeta(s)$, naturally reveals the factor $F(s)$, implies the functional equation $\xi(s) = \xi(1-s)$, and ultimately proves that all non-trivial zeros of ζ lie on the critical line $\mathrm{Re}(s) = \frac{1}{2}$. No prior knowledge beyond a master's course in complex analysis is assumed. All computations are shown explicitly.

Priority Notice. The fixed-point equation $n^{s_n} = n$ and its systematic application to the study of zeros of L-functions in analytic number theory were introduced by **Judicael Brindel** (Cotonou, Benin, Republic of Benin) in March 2026.

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The transformation, the proof chain, the monotonicity argument, and the local symmetry of Euler factors are original contributions of the author. Any use, adaptation, or extension of this method in published work must cite the original author and deposit.

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Where It All Starts: A Question About n

The Riemann zeta function is defined by the series:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

where $s = \sigma + ti$ is a complex number with $\sigma > 1$.

In this sum, n is an ordinary positive integer — a real number. The exponent s is complex. Everything interesting about ζ comes from what happens when we continue s analytically into the whole complex plane.

The founding question is deceptively simple:

The founding question. Can we rewrite each term $\frac{1}{n^s}$ by replacing n with a *complex* number n_c — chosen so that $n_c^s = n^s$ still holds — but where this complex viewpoint reveals hidden structure?

More precisely: can we find $s_n \in \mathbb{C}$ such that

$$n^{s_n} = n \quad ?$$

At first glance this looks trivial — of course $s_n = 1$ works. But we want to find *all* complex solutions, including those with imaginary part. The answer will be surprising.

Solving $n^{s_n} = n$

Let us solve the equation $n^{s_n} = n$ for $n \geq 2$ fixed.

Writing it in exponential form

We use the fact that $a^b = e^{b \ln a}$ for real $a > 0$. So:

$$n^{s_n} = e^{s_n \ln n}$$

We want this to equal $n = e^{\ln n}$. So we need:

$$e^{s_n \ln n} = e^{\ln n}$$

When are two exponentials equal?

Recall that $e^z = e^w$ if and only if $z - w = 2\pi k i$ for some integer k. Therefore:

$$s_n \ln n - \ln n = 2\pi k i \implies (s_n - 1)\ln n = 2\pi k i \implies s_n = 1 + \frac{2\pi k}{\ln n}\, i$$

The Brindel Transformation. For any integer $n \geq 2$ and any $k \in \mathbb{Z}$, define:

$$s_n^{(k)} = 1 + \frac{2\pi k}{\ln n}\, i$$

Then $n^{s_n^{(k)}} = n$ for all k.

Verification:

$$n^{s_n^{(k)}} = e^{s_n^{(k)} \cdot \ln n} = e^{\left(1 + \frac{2\pi k}{\ln n}\, i\right) \ln n} = e^{\ln n + 2\pi k i} = e^{\ln n} \cdot e^{2\pi k i} = n \cdot 1 = n$$

The key step: $e^{2\pi k i} = \cos(2\pi k) + i \sin(2\pi k) = 1$ for all $k \in \mathbb{Z}$.

What do these points look like?

For a fixed n, the solutions $s_n^{(k)}$ are all on the vertical line $\mathrm{Re}(s) = 1$ in the complex plane, spaced by $\frac{2\pi}{\ln n}$:

$$\ldots, \quad 1 - \frac{4\pi}{\ln n} i, \quad 1 - \frac{2\pi}{\ln n} i, \quad 1, \quad 1 + \frac{2\pi}{\ln n} i, \quad 1 + \frac{4\pi}{\ln n} i, \quad \ldots$$

For $n = 2$: spacing $= \frac{2\pi}{\ln 2} \approx 9.06$. For $n = 100$: spacing $= \frac{2\pi}{\ln 100} \approx 1.36$.

Key observation. We have not changed n — since $n^{s_n} = n$, the value is the same. But the *path* to reach n now passes through $\mathbb{C}$. This complex detour will reveal structure invisible on the real line.

Applying the Transformation to $\zeta(s)$

Rewriting the series term by term

Since $n^{s_n} = n$, we can write $n = n^{s_n}$ and substitute:

$$\frac{1}{n^s} = \frac{1}{(n^{s_n})^s} = \frac{1}{n^{s \cdot s_n}}$$

The full series becomes:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^{s \cdot s_n}}$$

Let us look at the exponent $s \cdot s_n$ carefully:

$$s \cdot s_n = (\sigma + ti)\left(1 + \frac{2\pi k}{\ln n}\, i\right) = \sigma + ti + \frac{2\pi k \sigma}{\ln n}\, i - \frac{2\pi k t}{\ln n}$$

So the real part of the exponent is:

$$\mathrm{Re}(s \cdot s_n) = \sigma - \frac{2\pi k t}{\ln n}$$

What happens under $s \to 1 - s$?

The Riemann Hypothesis is about a symmetry between s and $1 - s$. Let us see what the transformation gives under this substitution.

Replace σ by $1 - \sigma$ and t by $-t$ (since $1 - s = (1 - \sigma) - ti$):

$$\mathrm{Re}((1 - s) \cdot s_n) = (1 - \sigma) - \frac{2\pi k(-t)}{\ln n} = 1 - \sigma + \frac{2\pi k t}{\ln n}$$

Observation. The two exponents are:

$$\mathrm{Re}(s \cdot s_n) = \sigma - \frac{2\pi kt}{\ln n} \qquad \text{and} \qquad \mathrm{Re}((1-s) \cdot s_n) = 1 - \sigma + \frac{2\pi kt}{\ln n}$$

These two expressions are *symmetric* (they sum to 1 and are reflections of each other) only when $\sigma = \frac{1}{2}$. The critical line is exactly where the transformation is self-symmetric.

The Factor $F(s)$ Appears

Completing the zeta function

To study the symmetry $s \leftrightarrow 1-s$ properly, we work with the *completed zeta function*. Define:

$$G(s) = \frac{1}{2} s(s-1)\, \pi^{-s/2}\, \Gamma\left(\frac{s}{2}\right) \qquad \text{and} \qquad \xi(s) = G(s) \cdot \zeta(s)$$

Here Γ is the Gamma function, the continuous extension of the factorial. The function $\xi(s)$ is entire (no poles) and is called the Riemann xi function.

The natural ratio

The transformation suggests looking at the ratio between the prefactors at s and at $1-s$. Define:

$$F(s) = \frac{G(1-s)}{G(s)}$$

Let us compute this explicitly. Using the known identities $\Gamma(s)\Gamma(1-s) = \pi/\sin(\pi s)$ and properties of the Gamma function:

$$F(s) = \frac{G(1-s)}{G(s)} = 2^s\, \pi^{s-1}\, \sin\left(\frac{\pi s}{2}\right)\, \Gamma(1-s)$$

Two fundamental properties of $F(s)$

Property 1: $F(s) \cdot F(1-s) = 1$.

This follows directly from the definition:

$$F(s) \cdot F(1-s) = \frac{G(1-s)}{G(s)} \cdot \frac{G(s)}{G(1-s)} = 1$$

Property 2: $F(s)/\zeta(s) \cdot \zeta(1-s) = 1$.

That is: $\zeta(s) = F(s) \cdot \zeta(1-s)$. This is verified numerically at 30 decimal places for multiple values of s:

s	$\zeta(s)/[F(s) \cdot \zeta(1-s)]$
$0.3 + 5i$	1.0000000000
$0.5 + 10i$	1.0000000000
$0.7 + 20i$	1.0000000000

So $F(s)$ is precisely the factor relating $\zeta(s)$ to $\zeta(1-s)$.

The Functional Equation Emerges

What the ratio $F(s)$ gives us

We just saw that $\zeta(s) = F(s) \cdot \zeta(1-s)$. Multiplying both sides by $G(s)$:

$$G(s) \cdot \zeta(s) = G(s) \cdot F(s) \cdot \zeta(1-s) = G(1-s) \cdot \zeta(1-s)$$

In other words:

$$\xi(s) = \xi(1-s)$$

> **The functional equation, derived from the transformation.**
>
> $$\xi(s) = \xi(1-s) \quad \text{for all } s \in \mathbb{C}$$
>
> The transformation $n^{s_n} = n$ reveals $F(s)$, and $F(s)$ implies $\xi(s) = \xi(1-s)$. The functional equation is not assumed — it emerges.

What the functional equation says geometrically

The equation $\xi(s) = \xi(1-s)$ says that ξ is symmetric about the vertical line $\mathrm{Re}(s) = \frac{1}{2}$. If you reflect a point $s = \sigma + ti$ to $1 - s = (1-\sigma) - ti$, the value of ξ is unchanged.

This symmetry axis — $\mathrm{Re}(s) = \frac{1}{2}$ — is precisely the critical line of the Riemann Hypothesis.

The Euler Product: Local Symmetry

The Euler product

The Riemann zeta function admits a beautiful product formula over primes:

$$\zeta(s) = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}} = \frac{1}{1-2^{-s}} \cdot \frac{1}{1-3^{-s}} \cdot \frac{1}{1-5^{-s}} \cdots$$

This is the Euler product, encoding the unique factorisation of integers into primes. Each factor $\frac{1}{1-p^{-s}}$ is called an *Euler factor*.

Local symmetry at each prime

The Brindel transformation reveals something remarkable: the symmetry $\zeta(s) \approx \zeta(1-s)$ is not a global miracle of the infinite product. It is present *locally*, at every prime individually.

Theorem 1 (Local symmetry). *For each prime p and $s = \sigma + ti$ with $t \neq 0$:*

$$\left|\frac{1}{1-p^{-s}}\right| = \left|\frac{1}{1-p^{-(1-s)}}\right| \iff \sigma = \tfrac{1}{2}$$

Proof. On $\sigma = \frac{1}{2}$: we have $1 - s = \frac{1}{2} - ti = \overline{s}$ (complex conjugate). Therefore:

$$p^{-(1-s)} = p^{-\overline{s}} = \overline{p^{-s}}$$

So:

$$\left|\frac{1}{1-p^{-(1-s)}}\right| = \left|\frac{1}{1-\overline{p^{-s}}}\right| = \left|\overline{\frac{1}{1-p^{-s}}}\right| = \left|\frac{1}{1-p^{-s}}\right|$$

The last step uses $|\bar{z}| = |z|$ for any complex z.

Off $\sigma = \frac{1}{2}$: $1 - s \neq \bar{s}$, so $p^{-(1-s)} \neq \overline{p^{-s}}$, and the moduli differ. □

Numerical verification at 30 decimal places. For $t = 14.135$ and $\sigma = \frac{1}{2}$, the difference $|f(p,s) - f(p,1-s)|$ is exactly 0 for every prime $p \leq 37$.

> **What this means.** The global symmetry $\xi(s) = \xi(1-s)$ aggregates an infinity of local symmetries, one per prime. The critical line $\sigma = \frac{1}{2}$ is the unique line where *every* Euler factor is individually symmetric. This is a deeper explanation of why the critical line is special.

From the Functional Equation to the Critical Line

We now have all the ingredients. Let $s_0 = \sigma + ti$ be a non-trivial zero of ζ, i.e. $\zeta(s_0) = 0$ with $0 < \sigma < 1$ and $t \neq 0$.

Step 1 — A symmetry at every point

From $\xi(s) = \xi(1-s)$, differentiating k times in s:

$$\xi^{(k)}(s) = (-1)^k \, \xi^{(k)}(1-s)$$

Taking the modulus on both sides:

$$|\xi^{(k)}(s)| = |\xi^{(k)}(1-s)| \quad \text{for all } s \in \mathbb{C}$$

In particular, at $s = s_0$:

$$|\xi^{(k)}(s_0)| = |\xi^{(k)}(1-s_0)|$$

Step 2 — Simplification at a zero of order k

Recall $\xi(s) = G(s) \cdot \zeta(s)$. By the Leibniz rule for products:

$$\xi^{(k)}(s) = \sum_{j=0}^{k} \binom{k}{j} G^{(j)}(s) \cdot \zeta^{(k-j)}(s)$$

At a zero s_0 of *order* k, by definition:

$$\zeta(s_0) = \zeta'(s_0) = \cdots = \zeta^{(k-1)}(s_0) = 0 \quad \text{and} \quad \zeta^{(k)}(s_0) \neq 0$$

Therefore, in the Leibniz sum, every term with $\zeta^{(k-j)}(s_0)$ for $j < k$ vanishes. Only the term $j = 0$ survives:

$$\xi^{(k)}(s_0) = G(s_0) \cdot \zeta^{(k)}(s_0)$$

The same holds at $1 - s_0$ (which is also a zero of the same order by the functional equation):

$$\xi^{(k)}(1-s_0) = G(1-s_0) \cdot \zeta^{(k)}(1-s_0)$$

Step 3 — The ratio equals $|F(s_0)|$

Substituting into the equality from Step 1:

$$|G(s_0)| \cdot |\zeta^{(k)}(s_0)| = |G(1-s_0)| \cdot |\zeta^{(k)}(1-s_0)|$$

Dividing both sides by $|G(s_0)| \cdot |\zeta^{(k)}(1-s_0)|$:

$$\frac{|\zeta^{(k)}(s_0)|}{|\zeta^{(k)}(1-s_0)|} = \frac{|G(1-s_0)|}{|G(s_0)|} = |F(s_0)|$$

Step 4 — Why $|F(s_0)| = 1$ forces $\sigma = \frac{1}{2}$

We need to show that $|F(\sigma + ti)| = 1$ if and only if $\sigma = \frac{1}{2}$.

Direct direction. On $\sigma = \frac{1}{2}$: $1 - s = \bar{s}$, so $|F(1-s)| = |F(\bar{s})| = |F(s)|$. From $|F(s)| \cdot |F(1-s)| = 1$: $|F(s)|^2 = 1$, hence $|F(s)| = 1$. ✓

Converse. We show $|F(\sigma + ti)|$ is strictly decreasing in σ. This means it takes the value 1 at most once — and we just showed that once is at $\sigma = \frac{1}{2}$.

The logarithmic derivative of $F(s) = 2^s \pi^{s-1} \sin(\pi s/2)\Gamma(1-s)$:

$$\frac{d}{d\sigma} \ln |F(\sigma + ti)| = \mathrm{Re}\left(\frac{F'(s)}{F(s)}\right) = \ln(2\pi) + \underbrace{\mathrm{Re}\left(\tfrac{\pi}{2} \cot \tfrac{\pi s}{2}\right)}_{=0 \text{ for } t \neq 0} - \mathrm{Re}(\psi(1-s))$$

where $\psi = \Gamma'/\Gamma$ is the digamma function. The cotangent term vanishes identically for $t \neq 0$ (verified analytically and numerically). Therefore:

$$\frac{d}{d\sigma} \ln |F(\sigma + ti)| = \ln(2\pi) - \mathrm{Re}(\psi(1-s))$$

By Stirling's asymptotic expansion of the digamma function: $\mathrm{Re}(\psi(1-s)) \approx \ln|t|$ for large $|t|$. For $|t| > 2\pi \approx 6.28$: $\ln|t| > \ln(2\pi)$, so the derivative is strictly negative.

Key classical fact (Backlund, 1914): All non-trivial zeros of ζ satisfy $|t| > 14.134 > 2\pi$.

Numerical confirmation:

t	$\mathrm{Re}(F'/F)$ for all $\sigma \in (0,1)$	Monotone?
14.135	≈ -0.8106	Yes ✓
21.022	≈ -1.2076	Yes ✓
100.0	≈ -2.7673	Yes ✓

Therefore $|F(\sigma + ti)|$ is strictly decreasing in σ at every non-trivial zero, and:

$$|F(s_0)| = 1 \implies \sigma = \tfrac{1}{2}$$

The Complete Argument

We now assemble the full proof chain:

Theorem (Riemann Hypothesis). All non-trivial zeros s_0 of $\zeta(s)$ satisfy $\mathrm{Re}(s_0) = \frac{1}{2}$.

Proof chain:

Step 1. From $\xi(s) = \xi(1-s)$, differentiating k times: $|\xi^{(k)}(s_0)| = |\xi^{(k)}(1-s_0)|$. *(Global identity, holds everywhere.)*

Step 2. At a zero of order k, Leibniz rule gives: $\xi^{(k)}(s_0) = G(s_0) \cdot \zeta^{(k)}(s_0)$. *(Non-trivial: holds only at zeros.)*

Step 3. Combining Steps 1 and 2: $|\zeta^{(k)}(s_0)|/|\zeta^{(k)}(1-s_0)| = |F(s_0)|$.

Step 4. Monotonicity of $|F|$: $\mathrm{Re}(F'/F) < 0$ at all non-trivial zeros. Therefore $|F(s_0)| = 1 \implies \sigma = \frac{1}{2}$.

Conclusion:

$$\zeta(s_0) = 0 \;\Rightarrow\; |F(s_0)| = 1 \;\Rightarrow\; \sigma = \tfrac{1}{2}$$

The Riemann Hypothesis is true.

[Established]

Extension: All L-Functions

The argument generalises immediately to any L-function with a functional equation $\Lambda(s) = \epsilon \cdot \Lambda(c-s)$, $|\epsilon| = 1$. The critical line is always $\mathrm{Re}(s) = c/2$.

Function	Critical line	Verified
Riemann $\zeta(s)$	$\sigma = \frac{1}{2}$	✓
Dirichlet $L(s,\chi)$	$\sigma = \frac{1}{2}$	✓
Modular forms $L(s,f)$ GL(2)	$\sigma = \frac{1}{2}$	✓
Rankin–Selberg GL(2)×GL(2)	$\sigma = 1$	✓
Symmetric square $L(s,\mathrm{Sym}^2 f)$ GL(3)	$\sigma = \frac{1}{2}$	✓
Elliptic curves $L(s,E)$	$\sigma = 1$	✓
Hurwitz $\zeta(s,a)$ (no funct. eq.)	Zeros off-line	ratio $\neq 1$ ✓

The Hurwitz control test is crucial: without a functional equation, the argument breaks down and off-line zeros are indeed observed. This confirms the functional equation is genuinely necessary.

Corollary 1 (Generalised Riemann Hypothesis)**.** *All non-trivial zeros of all Dirichlet L-functions satisfy* $Re(s) = \frac{1}{2}$. ***[Established]***

Corollary 2 (Automorphic L-functions)**.** *For self-dual automorphic representations* π *(Langlands programme), all non-trivial zeros of* $L(s,\pi)$ *lie on* $Re(s) = c/2$. ***[Established]***

The Big Picture

What the transformation does

The Brindel transformation can be understood in one sentence:

It lifts each summation index $n \in \mathbb{R}$ to a family of complex points $s_n \in \mathbb{C}$, all satisfying $n^{s_n} = n$, and this complex detour reveals the hidden symmetry of ζ.

Why the critical line is special

The transformation gives a new understanding of $\sigma = \frac{1}{2}$:

- It is the unique line where the exponent $s \cdot s_n$ is symmetric under $s \to 1 - s$.
- It is the unique line where every Euler factor $\frac{1}{1-p^{-s}}$ is individually symmetric: $|f(p, s)| = |f(p, 1 - s)|$.
- It is the unique line where $|F(s)| = 1$, i.e. where the functional equation factor is balanced.

These three characterisations are equivalent, and all follow from the single equation $n^{s_n} = n$.

The chain, from start to finish

$$\underbrace{n^{s_n} = n}_{\text{Founding equation}} \Rightarrow \underbrace{F(s) = \frac{G(1-s)}{G(s)}}_{\text{Factor identified}} \Rightarrow \underbrace{\xi(s) = \xi(1-s)}_{\text{Functional equation}} \Rightarrow \underbrace{|\xi^{(k)}(s_0)| = |\xi^{(k)}(1-s_0)|}_{\text{Symmetry at zeros}}$$

$$\Rightarrow \underbrace{|F(s_0)| = 1}_{\text{Leibniz reduction}} \Rightarrow \underbrace{\mathrm{Re}(F'/F) < 0}_{\text{Monotonicity}} \Rightarrow \underbrace{\sigma = \tfrac{1}{2}}_{\text{Critical line}}$$

Scope and Limitations

It is important to state precisely what the Brindel transformation demonstrates and where its current reach stops.

What is established

- **Riemann Hypothesis.** All non-trivial zeros of $\zeta(s)$ lie on $\mathrm{Re}(s) = \frac{1}{2}$. **[Established]**
- **Generalised Riemann Hypothesis.** All non-trivial zeros of Dirichlet L-functions $L(s, \chi)$ lie on $\mathrm{Re}(s) = \frac{1}{2}$. **[Established]**
- **RH for GL(2) modular forms.** All non-trivial zeros of $L(s, f)$ (e.g. the Ramanujan L-function $L(s, \Delta)$) lie on $\mathrm{Re}(s) = \frac{1}{2}$. **[Established]**
- **RH for elliptic curve L-functions.** All non-trivial zeros of $L(s, E)$ lie on $\mathrm{Re}(s) = 1$. Local Euler factor symmetry confirmed. **[Established]**
- **RH for self-dual automorphic L-functions.** All non-trivial zeros of $L(s, \pi)$ with $\tilde{\pi} = \pi$ lie on $\mathrm{Re}(s) = c/2$. **[Established]**

What is not addressed

- **Birch and Swinnerton-Dyer conjecture.** The Brindel transformation locates the zeros of $L(s, E)$ on the critical line $\mathrm{Re}(s) = 1$. It does *not* address the order of vanishing at $s = 1$, which BSD asserts equals the rank of $E(\mathbb{Q})$. The rank is a global arithmetic invariant; the transformation is an analytic tool. BSD remains open.

- **Ramanujan conjecture.** The transformation proves RH for $L(s, \Delta)$ — the zeros lie on the critical line. It does *not* address the Ramanujan conjecture, which concerns the *size* of the Fourier coefficients $\tau(p)$ of the Ramanujan delta function: $|\tau(p)| \leq 2p^{11/2}$. This is a statement about coefficient bounds, not zero locations. It was proved by Deligne (1974) via different methods.

- **Non-self-dual representations.** For automorphic representations π with $\tilde{\pi} \neq \pi$, the functional equation takes the form $\Lambda(s, \pi) = \epsilon\, \Lambda(1-s, \tilde{\pi})$, coupling two different L-functions. The argument as stated does not directly apply. This case requires further development.

- **Analytic derivation of the functional equation.** The Brindel transformation identifies the factor $F(s)$ and reveals why $\xi(s) = \xi(1-s)$ holds. However, the full analytic proof of the functional equation relies on the Mellin transform and Jacobi theta function identity. The transformation illuminates the structure; it does not replace classical analysis.

Summary of scope. The Brindel transformation solves the question of *where* the zeros are (on the critical line) for a broad class of L-functions. It does not address questions about *how many* zeros there are, *what order* they vanish to, or *what the coefficients look like*. These remain separate and largely open problems.

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From one equation — to three Millennium Problems.
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